

Non-equilibrium Critical Dynamics and Precursory Phenomena in Color Superconductivity

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We derive an effective equation to describe the non-equilibrium dynamics of the pair field of color superconductivity near the critical temperature. We also discuss how critical dynamics affects the specific heat and density of state above the critical temperature.

The strong coupling nature of QCD at low energy can enhance the strength of the fluctuation of the pair field in the color superconductivity (CS) at low density around the critical temperature T_c . In fact, it was shown that there appear large pair fluctuations in rather wide range of temperatures near T_c .¹⁾ In this talk, we first derive an effective equation to describe the non-equilibrium critical dynamics of the pair field.²⁾ Then, we consider effects of the pair fluctuation on some observables and show that anomalous behaviors are clearly seen in these observables even well above T_c .^{2),3)}

Since we are interested in relatively low energy regions, we shall employ the Nambu-Jona-Lasinio model with two-flavors as an low energy effective theory of QCD. In the random phase approximation, the response function of the pair field reads $D^R(\mathbf{k}, \omega) = -Q(\mathbf{k}, \omega) / (G_C^{-1} + Q(\mathbf{k}, \omega))$, with $Q(\mathbf{k}, \omega)$ denoting the lowest polarization function.¹⁾ There appears anomalous behavior in D^R around T_c in accordance with the growth of the fluctuation of the diquark pair field. The large fluctuations in turn contribute to various observables. For example, effects on the quark self-energy and density of state near T_c is explored in 3) and shown that the *pseudogap* is formed above T_c as a precursory phenomenon of the CS (see, the left panel of Fig. 1). It will be also shown later that the pair fluctuation can affect the specific heat.²⁾

Before the calculation of observables, we first consider the non-equilibrium dynamics of the pair field near T_c . The dynamics of the pair field is well described by the pole of the collective mode (the *soft mode* of the CS) $\omega = \omega(\mathbf{k})$. Since the response function D^R diverges at $\omega = \omega(\mathbf{k})$, the collective mode is given by $\Xi^{-1} \equiv G_C^{-1} + Q(\mathbf{k}, \omega(\mathbf{k})) = 0$. Here, we expand Ξ^{-1} around $\omega = |\mathbf{k}| = 0$ and $T = T_c$:

$$\Xi^{-1}(\mathbf{k}, \omega) \equiv G_C^{-1} + Q(\mathbf{k}, \omega) \simeq A\epsilon + B|\mathbf{k}|^2 + C\omega, \quad (1)$$

where $\epsilon \equiv (T - T_c)/T_c$ is the reduced temperature and $A = T\partial Q(\mathbf{0}, 0)/\partial T|_{T=T_c}$,

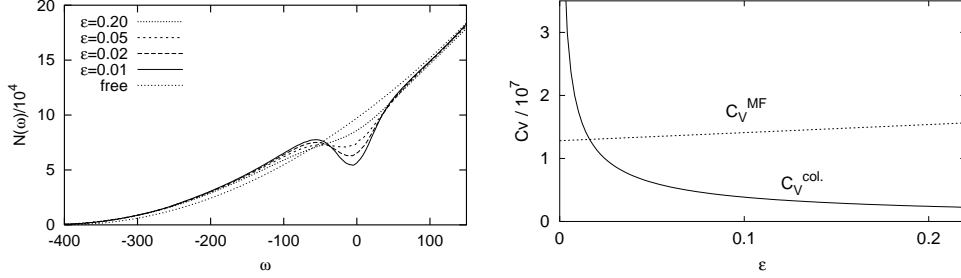


Fig. 1. **Left panel:** The density of state above T_c for various reduced temperature $\epsilon \equiv (T - T_c)/T_c$ with $\mu = 400\text{MeV}$.³⁾ **Right panel:** The specific heat C_v with $\mu = 400\text{MeV}$.²⁾ The contribution from the collective mode $C_v^{col.}$ is enhanced anomalously from $\epsilon = 0.05 \sim 0.1$.

$B = \partial Q(\mathbf{0}, 0)/\partial \mathbf{k}^2|_{T=T_c}$ and $C = \partial Q(\mathbf{0}, 0)/\partial \omega|_{T=T_c}$. Then, the pole of the soft mode $\Xi^{-1} = 0$ reads $\omega = -(A/C)\epsilon - (B/C)|\mathbf{k}|^2$. A numerical calculation shows that this simple equation well reproduces the position of pole in $T \lesssim 1.3T_c$ and $k \lesssim 150\text{MeV}$.²⁾

In the linear response theory, the dynamics of the pair field $\Delta(\mathbf{k}, \omega)$ is given by $\Xi(\mathbf{k}, \omega)\Delta(\mathbf{k}, \omega) = 0$;¹⁾ which, with expansion (1) corresponds to the linearized time-dependent Ginzburg-Landau equation of Δ .²⁾ The time evolution of Δ is easily calculated with this effective equation. It is worth mentioning that the coefficient C is not pure imaginary which means that the collective mode is a damped oscillating mode in this case. This is different from the soft mode in the weak coupling limit which is found to be an overdamped mode. One can show that the particle-hole asymmetry gives rise to the finite real part of C .²⁾

Next, we turn to the discussion on the specific heat C_v , which is given by the thermodynamic potential Ω ; $C_v = -T\partial^2\Omega/\partial T^2$. The contribution to Ω from the collective modes $\Omega_{col.}$ may be given by the summation of the connected ring diagrams,

$$\Omega_{col.} = 3T \sum_n \int \frac{d^3\mathbf{p}}{(2\pi)^3} \log(G_C^{-1} + Q(\mathbf{k}, i\nu_n)) = 3T \sum_n \int \frac{d^3\mathbf{p}}{(2\pi)^3} \log \Xi^{-1}(\mathbf{k}, i\nu_n).$$

Then, $C_v^{col.} = -T\partial^2\Omega_{col.}/\partial T^2$ is the contribution to the specific heat from the collective modes. Note that the calculation of $C_v^{col.}$ is well simplified using the expansion (1). Temperature dependence of $C_v^{col.}$ is shown in the right panel of Fig. 1 together with the specific heat of the free quark system C_v^{MF} ; the sum of them $C_v = C_v^{MF} + C_v^{col.}$ gives a total specific heat of the system. From the figure, one sees that $C_v^{col.}$ is enhanced anomalously as above T_c as $\epsilon = 0.05 \sim 0.1$. The enhancement of the specific heat in such a wide range of T might affect the cooling process of newly born compact stars. It is also interesting to investigate other precursory phenomena in various observables.

References

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